NONSTATIONARY ADIABATIC CENTRALLY-SYMMETRIC MOTIONS OF MATTER IN THE GENERAL THEORY OF RELATIVITY

(NESTATSIONARNYE ADIABATICHESKIE TSENTRAL'NO-SIMMETRICHNYE DVIZHENIIA MATERII V OBSHCHEI TEORII OTNOSITEL'NOSTI)

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1. Fundamental equations. In the case of unsteady motions in an intrinsic gravity field it is necessary to find the combined solution of the Einstein gravitational field equations and the equations of the conservation of momentum energy contained in them [1]

$$R_i^{\ k} - \tfrac{1}{2}\delta_i^{\ k} = \varkappa T_i^{\ k}$$

$$T_{i;\,k}^{\ \ k} = \frac{1}{\sqrt{-g}}\frac{\partial(\sqrt{-g}\,T_i^{\ k})}{\partial x^k} - \frac{T^{kl}}{2}\frac{\partial g_{kl}}{\partial x^i} = 0 \tag{1.1}$$

$$T_i^{\ k} = (p+\varepsilon)\,u_i u^k + \delta_i^{\ k} p = \frac{W}{v}u_i u^k + \delta_i^{\ k} p$$

$$\varkappa = \frac{8\pi G}{c^4}, \qquad W = (p+\varepsilon)\,v = E + pv$$

Here R_i^k is the curvature tensor, T_i^k the energy-momentum tensor, $\varkappa$ the Einstein gravitational constant, W the heat content per unit mass, v the specific volume, u_k the component of four-velocity.

If the analysis is limited to centrally-symmetric motions, the space-time metric may be selected in the following form:

$$ds^2 = e^{\nu}c^2dt^2 - e^{\lambda}r^2 - r^2(d\beta^2 + \sin^2\beta\, d\varphi^2)$$

$$g_{00} = -e^{\nu}, \quad g_{11} = e^{\lambda}, \quad g_{22} = r^2, \quad g_{33} = r^2\sin^2\beta \tag{1.2}$$

$$\sqrt{-g} = \exp\left[{}^1/_2(\nu+\lambda)\right] r^2\sin\beta$$

Henceforth, we shall study only radial flows, when $d\beta\,/\,dt = 0,\ d\varphi\,/\,dt =$ In this case the chronometrically-invariant velocity is given by Expression

$$a^r = \frac{1}{\sqrt{-g_{00}}}\frac{dr}{dt} = e^{-{}^1/_2\nu}\frac{dr}{dt}, \qquad a^2 = a_r a^r = g_{11}\left(\frac{dr}{dt}\right)^2 = e^{\lambda-\nu}\left(\frac{dr}{dt}\right)^2$$

Here

$$ds^2 = e^{\nu}(c\theta\, dt)^2 = (c\theta\, d\tau)^2, \quad \theta = \sqrt{1 - a^2/c^2}, \quad d\tau = e^{1/2\nu}$$

where $d\tau$ is an element of proper time, and the components of four-velocity take the form

$$u^0 = \frac{1}{\theta}\frac{dt}{d\tau} = \frac{1}{\theta} e^{-1/2\nu}, \qquad u_0 = g_{00}u^0 = \frac{1}{\theta} e^{1/2\nu}$$

$$u^1 = \frac{dr}{d\tau}\frac{1}{c\theta} = \frac{a}{c\theta} e^{-1/2\lambda}, \quad u_1 = g_{11}u^1 = \frac{a}{c\theta} e^{1/2\lambda}, \quad u_0u^0 + u_1u^1 = -1 \quad (1.3)$$

The equations of the conservation of the energy-momentum (1.1) yield the equations of motion and the continuity equation. Since $dW = Td\sigma + vdp$, (where T is the absolute temperature, and σ the entropy), then for the adiabatic processes considered, the equation of conservation of entropy

$$\frac{d(Wu^i)}{ds} + \frac{\partial W}{\partial x^i} = \frac{W}{2} u^k u^l \frac{\partial g_{kl}}{\partial x^i} + T\frac{\partial \sigma}{\partial x^i}, \quad \frac{\partial}{\partial x^k}\left(\frac{\sqrt{-g}\, u^k}{v}\right) = 0, \quad \frac{d\sigma}{ds} = 0 \quad (1.4)$$

must also be used.

The system of equations (1.4) is a complete system of conservation equations characterizing the adiabatic flows. Substituting the four-velocity components (1.3) here and taking into account that

$$d(\ln \sqrt{-g}) = \frac{d\lambda + d\nu}{2} + 2\frac{dr}{r} + \tan\beta\, d\beta$$

we obtain a system of equations of the hydrodynamics of radial flows in an intrinsic gravitational field (1.5)

$$\frac{1}{(c\theta)^2}\left(A\frac{\partial a}{\partial t} + a\frac{\partial a}{\partial r}\right) - \frac{\omega^2}{c^2}\left(\frac{\partial \ln v}{\partial r} + \frac{aA}{c^2}\frac{\partial \ln v}{\partial t}\right) + \frac{1}{2}\left(\frac{Aa}{c^2}\frac{\partial \lambda}{\partial t} + \frac{\partial \nu}{\partial r}\right) = \frac{\theta^2 T}{W}\frac{\partial \sigma}{\partial r}$$

$$-\left(A\frac{\partial \ln v}{\partial t} + a\frac{\partial \ln v}{\partial r}\right) + \frac{1}{\theta^2}\left(\frac{\partial a}{\partial r} + \frac{aA}{\partial r} + \frac{aA}{c^2}\frac{\partial a}{\partial t}\right) + $$

$$+ \frac{2a}{r} + \frac{1}{2}\left(A\frac{\partial \lambda}{\partial t} + a\frac{\partial \nu}{\partial r}\right) = 0$$

$$A\frac{\partial \sigma}{\partial t} + a\frac{\partial \sigma}{\partial r} = 0, \qquad \frac{\omega^2}{c^2} = -\left(\frac{\partial \ln v}{\partial \ln W}\right)_\sigma, \quad A = \exp\left(\frac{\lambda - \nu}{2}\right)$$

Let us note that the expression for the velocity a^* along the characteristics of these equations has the same form as in the special theory of relativity

$$a^* = A\left(\frac{dr}{dt}\right)^* = \left(\frac{dr}{d\tau_0}\right)^* = \frac{dl}{d\tau} = \frac{a \pm \omega}{1 \pm a\omega/c^2}, \quad dl = e^{1/2\lambda}\, dr, \quad d\tau_0 = \frac{dt}{A} \quad (1.6)$$

This is indeed completely natural since the presence of the gravitational field cannot change the local relationship between the chronometrically-invariant components of the three-dimensional velocities a^*, a, ω measured at each point r by means of the observer's clock at the same point. However, as follows from (1.6), the first characteristic on the scattering front ($\omega = 0$) turns out to be rectilinear in the variables l, τ which have physical meaning, while it is curvilinear in the r, t variables.

Let us write down the field equations

$$\frac{\partial (re^{-\lambda})}{\partial r} = 1 + \varkappa r^2 T_0{}^0 = 1 - \frac{\varkappa r^2}{\theta^2}\left(\varepsilon + p\,\frac{a^2}{c^2}\right)$$

$$A\,\frac{\partial (re^{-\lambda})}{\partial t} = -\varkappa c A T_0{}^1 = \frac{\varkappa a r^2}{\theta^2}(\varepsilon + p)$$

$$\left(1 + r\,\frac{\partial \nu}{\partial r}\right)e^{-\lambda} = 1 + \frac{\varkappa r^2}{\theta^2}\left(p + \varepsilon\,\frac{a^2}{c^2}\right) = 1 + \varkappa r^2 T_1{}^1 \qquad (1.7)$$

$$\varkappa T_2{}^2 = \varkappa T_3{}^3 = \varkappa p = \frac{e^{-\lambda}}{2}\left[\frac{\partial^2 \nu}{\partial r^2} + \frac{1}{2}\left(\frac{\partial \nu}{\partial r}\right)^2 + \right.$$
$$\left. + \frac{1}{2}\,\frac{\partial(\nu - \lambda)}{\partial r} - \frac{\partial \nu}{\partial r}\,\frac{\partial \lambda}{\partial r} - \frac{A}{c^2}\,\frac{\partial}{\partial t}\left(A\,\frac{\partial \lambda}{\partial t}\right)\right]$$

Only two of these equations are independent of (1.5). It is convenient to write these two equations as

$$A\,\frac{\partial \lambda}{\partial t} + a\,\frac{\partial \lambda}{\partial r} = -a\left(\frac{e^{\lambda} - 1}{r} + \varkappa p r e^{\lambda}\right), \quad A\left(1 + \frac{a^2}{c^2}\right)\frac{\partial \lambda}{\partial t} + a\,\frac{\partial(\lambda + \nu)}{\partial r} = 0 \quad (1.8)$$

Equations (1.5) and the first equation of (1.8) determine the solution

$$p = p(r, t), \quad \sigma = \sigma(r, t), \quad a = a(r, t), \quad \lambda = \lambda(r, t), \quad \nu = \nu(r, t)$$

for a given equation of state $p = p(\varepsilon)$.

1) In the static case $a = 0$, the second equation (1.8) yields $\partial\lambda/\partial t = 0$, and from (1.5) we have $\partial \ln v / dt = 0,\ d\sigma / dt = 0,$ therefore

$$\frac{dp}{dr} = -\frac{1}{2}(p + \varepsilon)\,\frac{d\nu}{dr} \qquad (1.9)$$

Furthermore, we have from (1.7)

$$\frac{d}{dr}(re^{-\lambda}) = 1 - \varkappa r^2\varepsilon, \qquad e^{-\lambda} r\,\frac{d\nu}{dr} = \varkappa r^2 p + 1 - e^{-\lambda}$$

Using these relationships we eliminate $\nu(r)$ from (1.9)

$$\frac{d}{dr}\left[\frac{r(p + \varepsilon)(1 + \varkappa p r^2)}{\{(p + \varepsilon) - 2r\,dp / dr\}}\right] = 1 - \varkappa r^2\varepsilon \qquad (1.10)$$

Let us note that for p = const the equation of state of the Einstein closed static model of the universe $\varepsilon + 3p = 0$ is obtained from (1.10).

We hence conclude that the Einstein model corresponds to the model of a star with constant negative pressure. From the viewpoint of the external observer, closeness of such a star means impossibility of intersection of the limits of the star with the geodesic line of any signal, however, the absence of limits is not at all reflected, in principle. Hence, the closed static model may be considered as a self-contained bounded non-Euclidean formation submerged in an external spatial background, and therefore, closedness in no way denotes the uniqueness of this model of the universe.

Solving (1.10), we determine $p(r)$, then $\lambda(r)$, $\nu(r)$, which solves the equilibrium problem completely.

2) Of considerable interest is the study of radial motions of an ideal fluid in a specified external gravitational field, for example, the external or internal Schwarzschild field, when $\lambda = \lambda(r)$, $\nu = \nu(r)$ are given functions of r. It is easy to see that in this case, by introducing the independent

variable $dr_1 = A dr$, Equations (1.5) are reduced to a form analogous to the hydrodynamics equations of the special theory of relativity, and differing only in the kind of free terms

$$\frac{1}{(c\theta)^2}\left(\frac{\partial a}{\partial t} + a\frac{\partial a}{\partial r_1}\right) - \frac{\omega^2}{c^2}\left(\frac{\partial \ln v}{\partial r_1} + \frac{a}{c^2}\frac{\partial \ln v}{\partial t}\right) + \frac{1}{2}\frac{d\nu}{dr_1} = \frac{T\theta^2}{W}\frac{\partial \sigma}{\partial r_1}$$

$$-\left(\frac{\partial \ln v}{\partial t} + a\frac{\partial \ln v}{\partial r_1}\right) + \frac{1}{\theta^2}\left(\frac{\partial a}{\partial r_1} + \frac{a}{c^2}\frac{\partial a}{\partial t}\right) + \frac{a}{2}\frac{d\nu}{dr_1} + \frac{2a}{r_1}\frac{r_1}{rA} = 0 \quad (1.11)$$

$$\frac{\partial \sigma}{\partial t} + a\frac{\partial \sigma}{\partial r_1} = 0$$

For the external Schwarzschild field

$$\nu + \lambda = 0, \quad \frac{d\lambda}{dr} + \frac{e^{\lambda} - 1}{r} = 0, \quad e^{-\lambda} = e^{\nu} = 1 - \frac{r_0}{r}, \; r_0 = \frac{2GM_0}{c^2}$$

hence

$$\frac{d\nu}{dr_1} = \frac{r_0}{r^2(1 - r_0/r^2)^2}$$

Here M_0 is the mass of the central body generating the field.

3) In the general case of adiabatic flows in an intrinsic gravitational field, finding the solution of the combined systems of Equations (1.5) and the first equation of (1.8) is of considerable difficulty. At the conclusion, we will obtain the solution in the asymptotic case of motions with velocities close to the velocity of light and of the ultra-relativistic equation of state. Here we propose a method of successive integration of (1.5) and the first equation of (1.8) by utilizing (1.7).

First of all, let us eliminate the function $\nu = \nu(r, t)$ from (1.5) by using the second equation of (1.8); we obtain

$$\frac{1}{(c\theta)^2}\left(A\frac{\partial a}{\partial t} + a\frac{\partial a}{\partial r}\right) - \frac{\omega^2}{c^2}\left(\frac{\partial \ln v}{\partial r} + \frac{aA}{c^2}\frac{\partial \ln v}{\partial t}\right) = \frac{1}{2a}\left(A\frac{\partial \lambda}{\partial t} + a\frac{\partial \lambda}{\partial r}\right) + \frac{\theta^2 T}{W}\frac{\partial \sigma}{\partial r}$$

$$-\left(A\frac{\partial \ln v}{\partial t} + a\frac{\partial \ln v}{\partial r}\right) + \frac{1}{\theta^2}\left(\frac{\partial a}{\partial r} + \frac{aA}{c^2}\frac{\partial a}{\partial t}\right) +$$

$$+\frac{2a}{r} = \frac{a}{2}\left(\frac{\partial \lambda}{\partial r} + \frac{aA}{c^2}\frac{\partial \lambda}{\partial t}\right) \quad (1.12)$$

Let us transform to the independent variables r, λ in (1.12); to do this, we find the Jacobian of the transformation $\partial(t; r)/\partial(\lambda; r)$ and also $\partial t/\partial r$ by using the first equation of (1.7) and the first equation of (1.8); we have

$$a\frac{\partial(t; r)}{\partial(\lambda; r)} = a\frac{\partial t}{\partial \lambda} = -\frac{AA_1 r}{A_4}, \qquad a\frac{\partial t}{\partial r} = A\left(1 - \frac{A_2}{A_4}\right) \quad (1.13)$$

Here

$$A_1 = e^{-\lambda}, \quad A_2 = 1 + \varkappa p r^2 - e^{-\lambda}, \quad A_3 = \varkappa r^2 \varepsilon + e^{-\lambda} - 1, \quad A_4 = \varkappa(p + \varepsilon)(r/\theta)^2$$

The relations (1.13) reduce Equations (1.12) to the symmetric form

$$\frac{1}{2(\theta c)^2}\left(A_1\frac{\partial a^2}{\partial \ln r}-A_2\frac{\partial a^2}{\partial \lambda}\right)-\frac{\omega^2}{c^2}\left(A_1\frac{\partial \ln v}{\partial \ln r}+A_3\frac{\partial \ln v}{\partial \lambda}\right)+\frac{1}{2}A_2=\frac{T\theta^2 A_4}{W}\frac{\partial \sigma}{\partial \lambda}$$

$$\left(A_1\frac{\partial \ln v}{\partial \ln r}-A_2\frac{\partial \ln v}{\partial \lambda}\right)-\frac{1}{\theta^2}\left(A_1\frac{\partial \ln v}{\partial \ln r}+A_3\frac{\partial \ln a}{\partial \lambda}\right)- \qquad (1.14)$$

$$-2A_1+\frac{A_3}{2}=0, \quad A_2\frac{\partial \sigma}{\partial \lambda}-A_1\frac{\partial \sigma}{\partial \ln r}=0$$

Let us recall that ω, ϵ and p, the functions v, σ, and Equations (1.4) contain three unknown functions

$$v=v(\lambda, r), \qquad a=a(\lambda, r), \qquad \sigma=\sigma(\lambda, r)$$

for the selected equation of state, say for $pv^k=\sigma$.

These equations may be integrated by the method of characteristics.

After this we determine $\nu=\nu(\lambda, r)$ from the first equation of (1.8), which becomes in the λ, r variables

$$A_1\frac{\partial \nu}{\partial r}+(A_4-A_2)\frac{\partial \nu}{\partial r}=\frac{a^2}{c^2}A_4+A_2 \qquad (1.15)$$

Finally, by using (1.13) we find $t=t(\lambda, r)$, which yields the complete solution of the problem.

Hence, the successive integration of (1.14), (1.13) and (1.15) permits the construction of the solution of isotropic motions not in an associated reference system where $a=0$, but in one connected with the isolated center of symmetry. The three-dimensional velocity a, measured in such reference systems, has a specific physical meaning in the formulation of boundary value problems both in a specified extrenal gravitational field, and in the study of motions in intrinsic gravitational fields.

Moreover, in the limiting case of no gravitational field (Galilean metric) Equations (1.5) transform into the hydrodynamic equations of the special theory of relativity, while such a transition is meaningless in an associated reference system since the latter is determined at once from the condition that the flux of energy-momentum equals zero. In the case of isentropic motions the system (1.14) reduces to two equations.

2. General solutions of particular cases of the equation of state.

a) D u s t l i k e   m a t e r i a l (*). In cosmological problems it is customarily assumed that the pressure is negligibly small as compared to the mean density of matter in the universe, i.e. $p \ll \varepsilon = \rho c^2$. If we put $p=0$, $\sigma=\text{const}$, then $\omega=0$, $d\ln v=-d\ln\epsilon$ and Equations (1.14) simplify radically

$$\frac{1}{(\theta c)^2}\left(\alpha\frac{\partial a^2}{\partial \lambda}-A_1\frac{\partial a^2}{\partial \ln r}\right)=\alpha, \qquad \alpha=1-A_1$$

$$A_1\frac{\partial \ln \varepsilon}{\partial \ln r}-\alpha\frac{\partial \ln \varepsilon}{\partial \lambda}+\frac{1}{\theta^2}\left(A_1\frac{\partial \ln a}{\partial \ln r}+A_3\frac{\partial \ln a}{\partial \lambda}\right)+2A_1-\frac{A_3}{2}=0 \qquad (2.1)$$

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*) R.Tollmann ([1], p.344) first solved the problem of motion of a gas with $p=0$ in the associated reference system. However, the velocities for the internal Schwarzschild problem were not evaluated in this solution.

The first equation of this system is easily integrated

$$\theta^2 = 1 - a^2 / c^2 = A_1 \Phi_1 (\alpha r) \tag{2.2}$$

Furthermore, we find from the second equation of (2.1)

$$\varepsilon = (\Phi_2(\alpha r) - B_2^{-1}) B_1, \qquad B_1 = \exp\left(\int B_3\, d\lambda\right), \qquad B_2 = \int B_4 B_1\, d\lambda,$$

$$B_3 = \alpha\left(\frac{1}{2\Phi_1} - \frac{3A_1+1}{2}\right) \tag{2.3}$$

$$B_4 = -\frac{\varkappa \alpha^3 r^2}{(1 - A_1\Phi_1)}\left\{\frac{r}{\Phi_1}\frac{d\Phi_1}{d(\alpha r)} + \frac{\Phi_1}{\alpha^2}(1 - A_1\Phi_1) - 1\right\}$$

Utilizing (2.2) and (2.3), we obtain for the function $\nu(r, \lambda)$ at $p = 0$ from Equation (1.15)

$$A_1 \frac{\partial \nu}{\partial r} + \frac{\partial \nu}{\partial \lambda}\left(\frac{\varkappa r \varepsilon}{\theta^2} - \frac{\alpha}{r}\right) = \frac{a^2 \varkappa r \varepsilon}{(c\theta)^2} + \frac{\alpha}{r} \tag{2.4}$$

This equation is also solved in quadratures. The equation of the characteristics

$$\frac{\partial \lambda}{\partial r} = e^{\lambda}\left(\frac{\varkappa r B_1 (\Phi_2 - B_2^{-1})}{\Phi_1} - \frac{\alpha}{r}\right)$$

yields $\chi_1(\lambda, r)$. The second integral determines the solution along the characteristics

$$\nu(\lambda, r) = \frac{\varkappa}{A_1}\int \frac{rB_1}{\Phi_1}(1 - A_1\Phi_1)(\Phi_2 - B_2^{-1})\, dr + \alpha \ln r + \chi(\chi_1(\lambda, r)) \tag{2.5}$$

Finally, the first equation of (1.13) gives the last quadrature

$$t = -\frac{1}{\varkappa r c}\int \frac{\Phi_1 \exp\{-1/2[\lambda + \nu(\lambda, r)]\}\, d\lambda}{\sqrt{1 - A_1\Phi_1(\Phi_2 - B_2^{-1}) B_1}} + \psi(r) \tag{2.6}$$

Hence, a general solution has been obtained for the problem of the motion of dustlike material, given by the integrals (2.2) to (2.6) and depending on the four arbitrary functions Φ_1, Φ_2, ψ, χ .

In solving concrete boundary value problems it is necessary to specify the initial velocity distribution $\theta_0 = \theta_0(\lambda, r)$ and to determine Φ_1 after which it is easy to find Φ_2, ψ, χ .

b) Ultra-relativistic approximation
Let us consider the case of adiabatic motions with velocities close to the velocity of light. Let us put $a/c = 1 - 2\Delta$, where $\Delta \ll 1$. Neglecting higher orders in Δ let us write the system (1.5) and (1.8) as

$$A\frac{\partial \ln(Wv)}{\partial t'} + \frac{\partial \ln(Wv)}{\partial r} = \frac{2}{r}, \qquad A\frac{\partial \lambda}{\partial t'} + \frac{\partial \lambda}{\partial r} = -\left[\frac{e^{\lambda} - 1}{r} + \varkappa p r e^{\lambda}\right]$$

$$A\frac{\partial \Pi}{\partial t'} + \frac{\partial \Pi}{\partial t'} = 0, \qquad A\frac{\partial \sigma}{\partial t'} + \frac{\partial \sigma}{\partial r} = 0, \; t' = ct, \; \Pi = \ln\Delta + \lambda - 2\ln W \tag{2.7}$$

Let us note that

$$d\ln(Wv) = \frac{\partial \ln(Wv)}{\partial p}\, dp + \frac{\partial \ln(Wv)}{\partial \sigma}\, d\sigma$$

The first equation of this system, taking the fourth into account, yields

$$A\frac{\partial p}{\partial t'}+\frac{\partial p}{\partial r}=\frac{2}{r}\left(\frac{\partial p}{\partial \ln (Wv)}\right)_\sigma \tag{2.8}$$

We transform the third equation of (2.7) analogously

$$A\frac{\partial (\ln\Delta+\lambda)}{\partial t'}+\frac{\partial (\ln\Delta+\lambda)}{\partial r}-2\left(\frac{\partial \ln W}{\partial p}\right)_\sigma\left(A\frac{\partial p}{\partial t'}+\frac{\partial p}{\partial r}\right)=0 \tag{2.9}$$

For the ultra-relativistic equation of state

$$p=(k-1)\varepsilon, \qquad Wv=v(\varepsilon v+pv)=\frac{kpv^2}{k-1}$$

Hence, taking into account that $pv^k=\sigma$ we find

$$\left(\frac{\partial \ln W}{\partial p}\right)_\sigma=\frac{k-1}{kp}, \qquad \left(\frac{\partial \ln (Wv)}{\partial p}\right)_\sigma=\frac{k-2}{kp}$$

Substituting these values into (2.7) and transforming to the independent variables p, r, we obtain the system

$$\begin{gathered}\frac{\partial\lambda}{\partial r}+\frac{e^\lambda-1}{r}+\varkappa rpe^\lambda-b\frac{\partial\lambda}{\partial p}=0\\ \frac{\partial(\ln\Delta+\lambda)}{\partial r}-b\frac{\partial(\ln\Delta+\lambda)}{\partial p}+\frac{4(k-1)}{(2-k)r}=0\\ \frac{\partial\sigma}{\partial r}-b\frac{\partial\sigma}{\partial p}=0, \quad A-\frac{\partial t'}{\partial r}+b\frac{\partial t'}{\partial p}=0, \quad b=\frac{2kp}{(2-k)p}\end{gathered} \tag{2.10}$$

The first equation is integrated at once

$$1-e^{-\lambda}=\frac{1}{r}F_1(\gamma)-c_1, \quad \gamma=pr^{2k/2-k}, \quad c_1=\frac{\varkappa pr^2(2-k)}{6-5k} \tag{2.11}$$

After this, it is easy to write the solution of the second equation of (2.10)

$$\Delta=e^{-\lambda}F_2(\gamma)p^{2(k-1)/k}=\left(1-\frac{1}{r}F_1+c_1\right)F_2p^{2(k-1)/k} \tag{2.12}$$

The third equation of (2.10) yields

$$\sigma=F_3(\gamma) \tag{2.13}$$

The fourth equation of (2.10) may be solved after determining $\nu=\nu(p,r)$ since $A=\exp[\frac{1}{2}(\lambda-\nu)]$ enters therein. To determine ν we use the second equation of (1.8) written in the p, r variables, where in our approximation

$$2A\frac{\partial\lambda}{\partial p}-\frac{\partial t'}{\partial r}\frac{\partial(\lambda+\nu)}{\partial p}+\frac{\partial t'}{\partial p}\frac{\partial(\lambda+\nu)}{\partial r}=0 \tag{2.14}$$

We form the derivative $\partial t'/\partial r$ from the fourth equation of (2.10), and we determine $\partial t'/\partial p$ from the second equation of (1.7)

$$2\frac{\partial\lambda}{\partial p}-\frac{\partial(\lambda+\nu)}{\partial p}=e^\lambda\frac{\partial\lambda}{\partial p}\frac{4(k-1)\Delta}{\varkappa kp^2}\left[\frac{\partial(\lambda+\nu)}{\partial r}-b\frac{\partial(\lambda+\nu)}{\partial p}\right]$$

This equation yields $\nu=\nu(p,r)$, after which we write the last quadrature for

$$ct=F_4(\gamma)+\int\exp\left[{}^1/_2\{\lambda(p,r)-\nu(p,r)\}\right]dr \tag{2.15}$$

The constructed solution depends on five arbitrary functions and solves the problem posed.

It can be said that the fundamental equations (1.5) and (1.7) (which permit inclusion even of electromagnetic fields in the consideration) completely describe centrally-symmetric radial flows in an intrinsic gravity field and may be ustilized in cosmology of isotropic space. The general theory of relativity is simply gas dynamics in Riemann space in this sence.

It should be noted that the problem posed here of investigating exact equations convenient for a description of the relativistic motion of a medium in an intrinsic gravity field, may be solved by using the variational methods of continua and the field equations [2].

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